CODE - A

TEST ID 002011

NTPL

ALL INDIA TEST SERIES

JEE (Main) - 2020

PART TEST - 1

Time : 3 Hours | ***Maximum Marks : 300***

Syllabus Covered

Physics : Error Analysis, Vector, Kinematics, Laws of Motion, Electrostatics.
Chemistry : Isomerism, Chemical Bonding, Stoichiometry, Redox, GOC.
Mathematics : Functions, Limits, Continuity and Differentiability, Application of Derivatives.

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

A. General :

1. This booklet is your Question Paper containing **75 questions**.
2. The Question Paper **CODE & TEST ID** is printed on the right hand top corner of this booklet. This should be entered on the OMR Sheet.
3. Fill the bubbles completely and properly using a **Blue/Black Ball Point Pen** only.
4. No additional sheets will be provided for rough work.
5. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers, and electronic gadgets in any form are not allowed to be carried inside the examination hall.
6. The answer sheet, a machine-readable Optical mark recognition sheet (OMR Sheet), is provided separately.
7. DO NOT TAMPER WITH / MUTILATE THE OMR OR THE BOOKLET.
8. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilator.

B. Question paper format & Marking Scheme :

9. The question paper consists of **3 parts** (Physics, Chemistry and Mathematics).
10. **Section I** contains **20 questions**. Each question has 4 choices (A), (B), (C) and (D), for its answer, out of which ***ONLY ONE*** is correct. Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.
11. **Section II** contains **5 questions**. The answer to each question is a ***NUMERICAL VALUE.*** Each question carries **+4 marks** for correct answer and **-1 mark** for wrong answer.

Name of the Candidate (in Capitals) ______________________________

Test Centre ____________________ Centre Code ____________________

Candidate's Signature ____________________ Invigilator's Signature ____________________

PART - I PHYSICS

SECTION 1 (Maximum Marks: 80)

This section contains **TWENTY (20)** questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

1. The resistance $R = \frac{V}{1}$ where $V = (100 \pm 5)$ volts and $i = (10 \pm 0.2)$ amperes. What is the total error in R?

(a) 5% (b) 7% (c) 5.2% (d) $\frac{5}{2}\%$

2. The period of oscillation of a simple pendulum in the experiment is recorded as 2.63s, 2.56s, 2.80s and 2.42s. The average absolute error is

(a) 0.11 s (b) 0.1 s (c) 0.01 s (d) 1.08

3. The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate by using the formula $p = \frac{F}{l^2}$. If the maximum errors in the measurement of force and length are 4% and 2% respectively, then the maximum error in the measurement of pressure is?

(a) 1% (b) 2% (c) 8% (d) 10%

4. A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 divisions above the reference level. If screw gauge has zero error of –0.004 cm, the correct diameter of the ball is

(a) 0.053 cm (b) 0.525 cm (c) 0.521 cm (d) 0.529 cm

5. Which of the following is a null vector?

(i) Velocity vector of a body moving in a circle with a uniform speed

(ii) Velocity vector of a body moving is a straight line with a uniform speed

(iii) Position vector of the origin of the rectangular coordinate system

(iv)Displacement vector of a stationary object

(a) Both *(i)* and *(ii)* (b) Both *(ii)* and *(iii)* (c) *(i), (ii)* and *(iii)* (d) *(iii)* and *(iv)*

6. If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, then *(i)* $\vec{A} = 0$ $\vec{B} = 0$, *(ii)* the angle between $\vec{A}$ and $\vec{B}$ is 90, *(iii)* Angle between $\vec{A}$ and $\vec{B}$ is 120º. Which of the following is possible?

(a) *(i)* and *(ii)* (b) *(ii)* and *(iii)* (c) *(i)* and *(iii)* (d) *(i), (ii)* and *(iii)*

7. The angle subtended by the vector, $\vec{A} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ with the $x-$axis is:

(a) $\sin^{-1}\left(\frac{3}{13}\right)$ (b) $\sin^{-1}\left(\frac{4}{13}\right)$ (c) $\cos^{-1}\left(\frac{4}{13}\right)$ (d) $\cos^{-1}\left(\frac{3}{13}\right)$

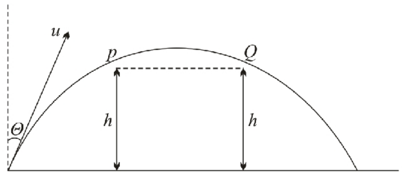
8. Given that $\vec{A}+\vec{B}=\vec{C}$ and $\vec{C}$ is $\perp r$ to $\vec{A}$. If $|\vec{A}|=|\vec{C}|$ then the angle between $\vec{A}$ and $\vec{B}$ is:

(a) $\pi/4$ (b) $\pi/2$ (c) π (d) $3\pi/4$

9. A particle starts from rest and travels a distance s with uniform acceleration, then it travels a distance 2s with uniform speed, finally it travels a distance 3s with uniform retardation and comes to rest. If the complete motion of the particle is a straight line then the ratio of its average velocity to maximum velocity is:

(a) 6/7 (b) 4/5 (c) 3/5 (d) 2/5

10. A particle is thrown with velocity u making an angle θ with the vertical. It just crosses the top of two poles each of height h after 1s and 3s respectively. The maximum height of projectile is



(a) 9.8 m (b) 19.6 m
(c) 39.2m (d) 4.9 m

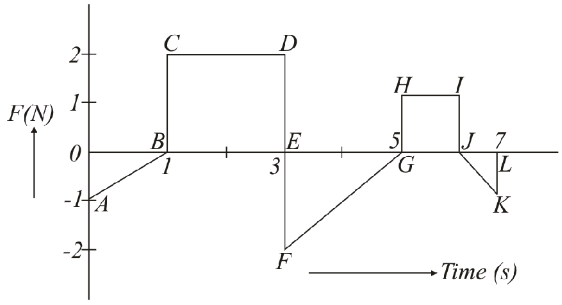
11. A body of mass m is moving on a circular path of radius r with a constant speed, u . The wok done by the centripetal force in moving the body over half the circumference of the circle is?

(a) $2mv^2$ (b) mv^2r (c) $mv^2/2r$ (d) zero

12. A projectile has a maximum range of 12 km. At the highest point of its motion, it explodes into two equal masses. One mass drops vertically downwards. The horizontal distance covered by the other mass from the time of explosion is:

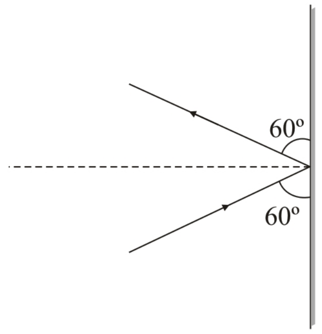
(a) 6 km (b) 12 km (c) 18 km (d) 24 km

13. A force time graph for linear motion of a body is shown in fig. The change in linear momentum between 0 and 7s is



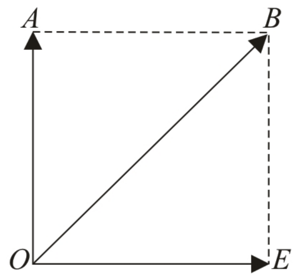
(a) 5 N-S
(b) 4 N-S
(c) 2 N-S
(d) 3 N-S

14. A 3 kg ball strikes a heavy rigid wall with a speed of 10 m/s at an angle of 60º. It gets reflected with the same speed at an angle shown in fig. If 0.20 s is the time of contact of the ball with the wall, the average force exerted by the ball on the wall is



(a) 150 N
(b) zero
(c) 300 N
(d) $150\sqrt{3}$N

15. A body of mass 2 kg has an initial velocity of 3 m/s along *OE*. It is subjected to a force to a force 4'N in a direction $\perp$ to OE. The displacement of body from *O* after 4 second will be

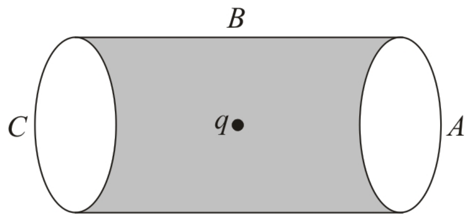


(a) 48 m
(b) 20 m
(c) 12 m
(d) 8 m

16. A car starts from rest to cover a distance s. The coefficient of friction between the rod and tyres is μ. The minimum time in which the car can cover the distance is proportional to

(a) μ (b) $\sqrt{\mu}$ (c) $\frac{1}{\sqrt{\mu}}$ (d) $\frac{1}{\mu}$

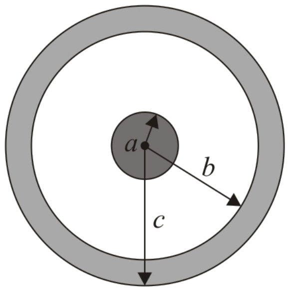
17. A hollow cylinder has a charge q coulomb within it. If ϕ is electric flux associated with the curved surface B. The flux linked with the plane surface A in same units will be



(a) $\frac{q}{2\in_0}$ (b) $\frac{\phi}{3}$

(c) $\frac{q}{\in_0}$ (d) $\frac{1}{2}\left(\frac{q}{\in_0}-\phi\right)$

18. A solid conducting sphere of radius a has a net positive charge 2 Q. A conducting spherical shell of linear radius b and outer radius c is concentric with the solid sphere and has net charge – Q. The surface charge density on the inner and outer surface of the spherical shell will be



(a) $-\frac{2Q}{4\pi b^2},\frac{Q}{4\pi c^2}$ (b) $-\frac{Q}{4\pi b^2},\frac{Q}{4\pi c^2}$

(c) $0,\frac{Q}{4\pi c^2}$ (d) None of these

19. A 100 eV electron is fired directly towards a large metal plate having surface charge density $-2\times10^{-6}cm^{-2}$. The distance from where the electron be projected so that it just fails to strike the plate is

(a) 0.22 mm (b) 0.44 mm (c) 0.66 mm (d) 0.88 mm

20. A small electric dipole is placed at origin with its dipole moment directed along positive x-axis. The direction of electric field at point $\left(2, 2\sqrt{2}, 0\right)$ is

(a) along negative $z-$axis
(b) along $z-$axis
(c) along negative $y-$axis
(d) along positive $y-$axis

SECTION 2 (Maximum Marks: 20)

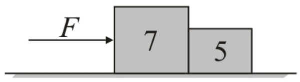
This section contains **FIVE (05)** questions. The answer to each question is a **NUMERICAL VALUE.** Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

21. $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} + 2\hat{j} + \hat{k}$. The component of $\vec{B}$ in the direction of $\vec{A}$ is $\frac{1}{n}$, then value of n is

22. A projectile is thrown in the upward direction making an angle of 60º with horizontal direction with a velocity of 150 ms^{-1}. Then the time after which its inclination with the horizontal is 45º, is (use $g = 10\,m/s^2$) $\left(\text{take } \sqrt{3} = 1.7\right)$

23. A body of mass 10 kg is lying on a rough plane inclined at an angle of 30º to the horizontal and the coefficient of friction is $\frac{1}{\sqrt{3}}$. The minimum force (in Newton) required to pull the body up the plane is ($g = 9.8 m/s^2$).

24. Two blocks of mass 7 kg and 5 kg are placed in contact with eachother on a smooth surface. If a force of 6 N is applied on the heavier block, what is the force on lighter block?



25. Charge 1c is distributed to two different metallic spheres having radii R and 2R such that both spheres have equal surface charge density. Then charge on larger sphere in columb is

PART - II CHEMISTRY

SECTION 1 (Maximum Marks: 80)

This section contains **TWENTY (20)** questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

26. According to Molecular Orbital Theory,

(a) C_2^{2-} is expected to be paramagnetic

(b) O_2^{2+} is expected to have a longer bond length than O_2

(c) N_2^+ and N_2^- have the same bond order

(d) He_2^+ has the same energy as two isolated He atoms

27. The species in which the N atom is in a state of *sp* hybridization is :

(a) NO_3^- (b) NO_2 (c) NO_2^+ (d) NO_2^-

28. Hydrogen bonding does not play a central role in the following phenomena:

(a) Ice floats in water.

(b) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions.

(c) Formic acid is more acidic than acetic acid.

(d) Dimerisation of acetic acid in benzene.

29. The molecule that will have dipole moment is:

(a) 2,2-dimethylpropane (b) *trans*-2-pentene

(c) *trans*-3-hexene (d) 2,2,3,3-tetramethylbutane

30. The non-linear structure is assumed by:

(a) $SnCl_2$ (b) NCO^- (c) CS_2 (d) NO_2^+

31. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is :

(a) 11.86 (b) 1186 (c) 84.3 (d) 118.6

32. The most abundant elements by mass in the body of a healthy human adult are : Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^{1}H atoms are replaced by ^{2}H atoms is :

(a) 10 kg (b) 15 kg (c) 37.5 kg (d) 7.5 kg

33. 8.4 g $MgCO_3$ on heating leaves behind a residue weighing 4.4 g carbon dioxide released into the atmosphere at S.T.P. will be –

(a) 2.24 L (b) 4.48 L (c) 1.12 L (d) 0.56 L

34. 5 moles of A and 6 moles of Z are mixed with sufficient amount of C to produce final product F. How many maximum moles of 'F' can be produced as per the given sequence of reaction ?

$A + 2Z \rightarrow B$; $B + C \rightarrow Z + F$

(a) 3 (b) 2 (c) 4 (d) 5

35. Equal masses of O_2, H_2 and CH_4 are taken in a container. The respective mole ratio of these gases in container is -

(a) 1 : 16 : 2 (b) 16 : 1 : 2 (c) 1 : 2 : 16 (d) 16 : 2 : 1

36. The oxidation number of sulphur in S_8, S_2F_2, H_2S respectively, are

(a) 0, +1 and –2 (b) +2, +1 and –2 (c) 0, +1 and +2 (d) –2, +1 and +2

37. For the redox reaction :

$$MnO_4^- + C_2O_4^{2-} + H^+ \longrightarrow Mn^{2+} + CO_2 + H_2O$$

the correct coefficients of the reactants for the balanced reaction are

	MnO_4^-	$C_2O_4^{2-}$	H^+
(a)	2	5	16
(b)	16	5	2
(c)	5	16	2
(d)	2	16	5

38. $AB_4^- + C^{+2} \rightarrow C^{+3} + A^{+2}$

If the O.N. of B is –2. Choose the true statement for the above change –

(a) O.N. of A decreases by –5

(b) O.N. of C decreases by +1

(c) O.N. of A decreases by + 5 and that of C increases by +1

(d) O.N. of A decreases by +5 and that of C decreases by +1

39. In which of the following reactions, the underlined element has decreased its oxidation number during the reaction?

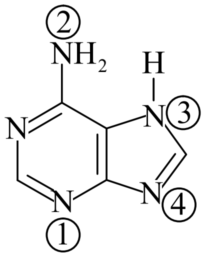
(a) $\underline{Fe} + CuSO_4 \rightarrow Cu + FeSO_4$ (b) $\underline{H_2} + Cl_2 \rightarrow 2HCl$

(c) $\underline{C} + H_2O \rightarrow CO + H_2$ (d) $\underline{Mn}O_2 + 4HCl \rightarrow MnCl_2 + Cl_2 + 2H_2O$

40. The oxidation state of Cr in CrO_5 is –

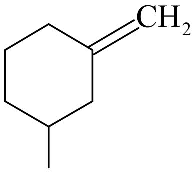
(a) +6 (b) +10 (c) +5 (d) +4

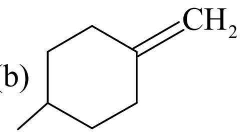
41. Which nitrogen in adenine is most basic?

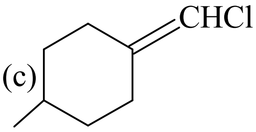


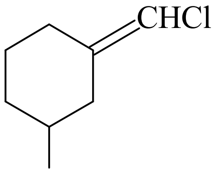
(a) 2 (b) 3 (c) 1 (d) 4

42. The geometrical isomerism is shown by

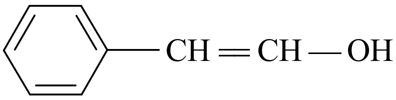
(a) 

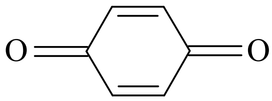
(b) 

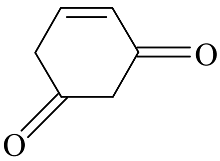
(c) 

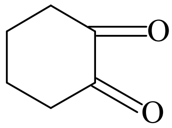
(d) 

43. Tautomerism is exhibited by

I. 

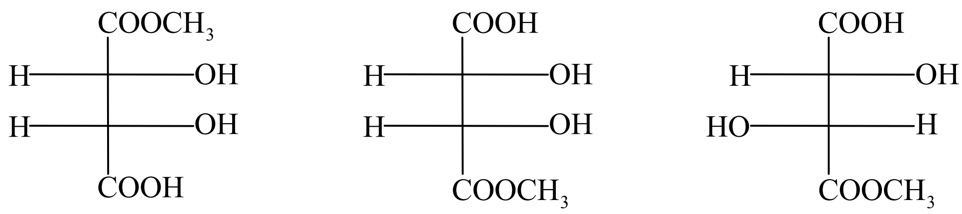
II. 

III. 

IV. 

(a) I, III and IV (b) II, III and IV (c) All the four (d) None of these

44. The correct statement about compound I, II, III is



I II III

(a) (I) and (III) are diastereomers
(b) (I) and (III) are enantiomers
(c) (I) and (II) are enantiomers
(d) (I) and (II) are identical.

45. The specific rotation of A is + 40º when A dissolves in solvent it is converted in (–) B. The specific rotation of equilibrium mixture is + 10º. The ratio of moles of A and B at equilibrium is

(a) 5:1
(b) 5:3
(c) 3:5
(d) 3:1

SECTION 2 (Maximum Marks: 20)

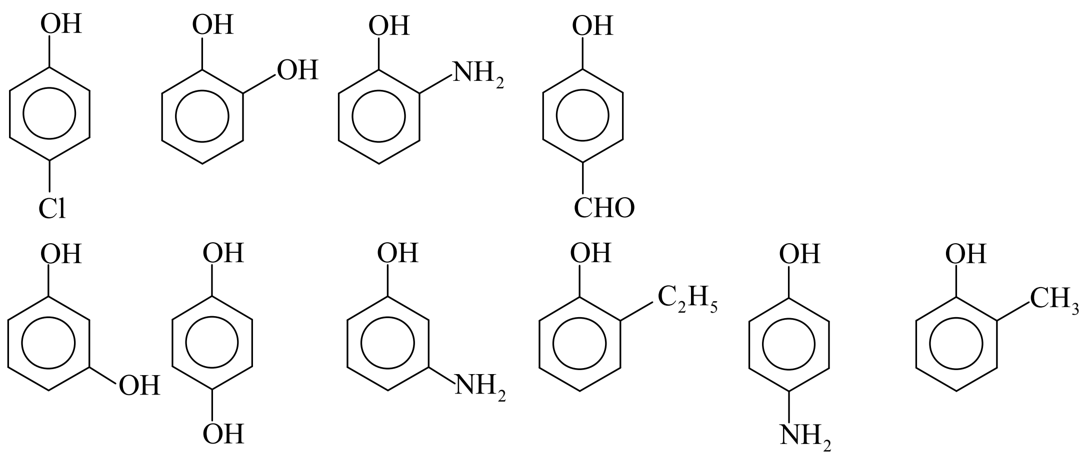
This section contains **FIVE (05)** questions. The answer to each question is a **NUMERICAL VALUE.** Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

46. Total number of isomer shown by molecule C_3H_6O

47. How many streo isomers can exist for following acid?

$$H-C(OH)COOH$$
$$H-C-CO_2H$$
$$H-C(OH)COOH$$

48. Among following numbers of molecule more acidic then phenols are



49. The number of isomer for the compound with molecular formula C_2BrClF is

50. The total number of cyclic isomer possible for a hydrocarbon with the molecular formula C_4H_6 is

PART - III MATHEMATICS

SECTION 1 (Maximum Marks: 80)

This section contains **TWENTY (20)** questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

51. Let $f: R \to R$, be defined as $f(x) = e^{x^2} + \cos x$, then f is :

(a) one-one and onto (b) one-one and into (c) many-one and onto (d) many-one and into

52. Let $f(x) = \min.(4x+1,\ x+2,\ -2x+4)$. Then the maximum value of $f(x)$ is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{8}{3}$

53. Let $f:(-1, 1) \to B$ be a function defined by $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then f is both one-one and onto when B is in the interval :

(a) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (c) $\left[0, \frac{\pi}{2}\right]$ (d) $\left(0, \frac{\pi}{2}\right)$

54. If $f(x) = 3x - 5$, then $f^{-1}(x)$:

(a) $\frac{1}{3x-5}$ (b) $\frac{x+5}{3}$

(c) does not exist because f is not one-one (d) does not exist because f is not onto

55. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the value of '$a$' and '$b$' are :

(a) $a \in R, b \in R$ (b) $a = 1, b \in R$ (c) $a \in R, b = 2$ (d) $a = 1, b = 2$

56. If $\lim_{x\to 0}\left(1 + x\ln\left(1+b^2\right)\right)^{1/x} = 2b\sin^2\theta,\ b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is

(a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

57. The value of $\lim_{x\to\infty}\frac{1}{x^6}\int_0^{x^2}\frac{t^6+5t^4+55}{2t^4+t+5}dt$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{12}$ (d) $\frac{1}{6}$

58. The value of p and q for which the function

$$f(x)=\begin{cases}\frac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$$

is continuous for all x in R, is :

(a) $p=\frac{5}{2}, q=\frac{1}{2}$ (b) $p=\frac{-3}{2}, q=\frac{1}{2}$ (c) $p=\frac{1}{2}, q=\frac{3}{2}$ (d) $p=\frac{1}{2}, q=\frac{-3}{2}$

59. Let $f(x)=\begin{cases}x^2, & x\in Z\\ \frac{k(x^2-4)}{2-x}, & x\notin Z\end{cases}$

where Z is the set of all integers, then $f(x)$ is continuous at $x=2$ for :

(a) $k=1$ only (b) every real k

(c) every real k except $k=-1$ (d) $k=-1$ only

60. If the function $f(x)=\begin{cases}x^2+px+1, & x \text{ is rational}\\ px^2+2x+q, & x \text{ is irrational}\end{cases}$ is continuous at $x=1$ and $x=2$, then $(p+q)$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$

61. Which of the following statement is true?

(a) The equation $\sin x - x = 0$ has a real root in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(b) The equation $\tan x - x = 0$ has a real root in $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

(c) If $f(x)$ is a real-valued continuous function in [0, 2], then there exist some $c \in R$ such that $f(x) \geq c$ for all $x \in [0, 2]$

(d) If $g(x)$ is a real-valued function defined on [3, 5] and $g(3) \cdot g(5) < 0$, then there exist some $\alpha \in (3, 5)$ such that $g(\alpha) = 0$

62. Let $f(x) = \begin{cases} x^2 e^{2(x-1)} & \text{for } 0 \leq x \leq 1 \\ a\,\text{sgn}(x+1)\cos(2x-2) + bx^2 & \text{for } 1 < x \leq 2 \end{cases}$

If $f(x)$ is differentiable at $x = 1$, then

(a) $a^2 + b^2 = 3$ (b) $a^2 - b^2 = 3$ (c) $ab = 2$ (c) $a - b = -3$

[Note : $\text{sgn}(k)$ denotes signum function of k]

63. Let $f(x) = \max.\{|x^2 - 2|x||, |x|\}$, then number of points where $f(x)$ is non-derivable, is :

(a) 2 (b) 3 (c) 4 (d) 5

64. Let $f(x) = \begin{cases} x+2, & x < 0 \\ -(2+x^2), & 0 \leq x < 1 \\ x, & x \geq 1 \end{cases}$. Then the number of points where $|f(x)|$ is non-derivable is :

(a) 3 (b) 2 (c) 1 (d) 0

65. The left hand derivatives of $f(x) = [x]\sin(\pi x)$ at $x = k$, k an integer, is :

(a) $(-1)^k (k-1)\pi$ (b) $(-1)^{k-1}(k-1)\pi$ (c) $(-1)^k k\pi$ (d) $(-1)^{k-1} k\pi$

66. Which of the following functions is differentiable at $x = 0$?

(a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$ (c) $\sin(|x|) + |x|$ (d) $\sin(|x|) - |x|$

67. If $g(x)=\begin{cases} 2-3x^2, & -\infty < x \le -1 \\ mx, & -1 < x < 0 \\ x^2, & 0 \le x < \infty \end{cases}$, then the range of m such that $g(x)$ is monotonically increasing in $x \in (-\infty, \infty)$ is :

(a) $[0, 1)$ (b) $(0, 1)$ (c) $[0, 1]$ (d) $(0, 1]$

68. If the curves $\frac{x^2}{c}+\frac{y^2}{4}=1$ and $y^3=16x$ intersect at right angles, then c is equal to :

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{4}{3}$ (d) 2

69. In which one of the following functions, Rolle's theorem is applicable?

(a) $f(x)=|x|$, in $-2 \le x \le 2$ (b) $f(x)=\tan x$, in $0 \le x \le \pi$

(c) $f(x)=1+(x-2)^{2/3}$, in $1 \le x \le 3$ (d) $f(x)=x(x-2)^2$, in $0 \le x \le 2$

70. Let $f(x)=\int_3^{x^2} e^{-t^2}(t-4)(t^2-t+12)\,dt$. An interval in which $f(x)$ is decreasing is :

(a) $(-2, 0)$ (b) $(0, 2)$ (c) $(2, \infty)$ (d) $(-1, 1)$

SECTION 2 (Maximum Marks: 20)

This section contains **FIVE (05)** questions. The answer to each question is a **NUMERICAL VALUE.** Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

71. The smallest positive integral value of $f(x)=\frac{x^2+x+7}{x+2}$, $x \in R$ is equal to :

72. If $[x]^2+[x-2]<0$ and $\{x\}=\frac{1}{2}$, then the number of possible values of x, is :

[**Note :** $[x]$ and $\{x\}$ denote greatest integer less than or equal to x and fractional part of x respectively]

73. Let $f: R \to R$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}$ is equal to :

74. Let $f(x)=\begin{cases}\dfrac{\left(x^3+x^2-16x+20\right)}{(x-2)^2}, & \text{if } x\neq 2\\ k, & \text{if } x=2\end{cases}$

If $f(x)$ is continuous for all x, then k is equal to :

75. Let $f(x)=\begin{cases}\left(\dfrac{x\sin x+2\cos 2x}{2}\right)^{\frac{1}{x^2}}, & x\neq 0\\ e^{\frac{-k}{2}}, & x=0\end{cases}$

If $f(x)$ is continuous at $x=0$, then the value of k is :

SOLUTION OF AITS JEE(MAIN) PART TEST – 1

PHYSICS

1. **(b)**

From $R=\frac{V}{I}$; $\frac{\Delta R}{R}=\frac{\Delta V}{V}+\frac{\Delta I}{I}=\frac{5}{100}+\frac{0.2}{10}=\frac{7}{100}$

$\therefore\ \frac{\Delta R}{R}\times 100=\frac{7}{100}\times 100=7\%$

2. **(a)**

Here $a_1=2.63s,\ a_2=2.56s,\ a_3=2.71s,$

$a_4=2.80s,\ a_5=2.42s$

$a_m=\frac{a_1+a_3+a_4+a_5}{5}$

$=\frac{2.63+2.56+2.71+2.80+2.42}{5}=\frac{13.12}{5}\ =2.62s$

$\Delta a_1=2.63-2.62=0.01;\ \Delta a_2=2.56-2.62=-0.06$

$\Delta a_3=2.71-2.62=0.09;\ \Delta a_4=2.80-2.62=0.18$

$\Delta a_5=2.42-2.62=-0.20$

Answer absolute error

$=\frac{|\Delta a_1|+|\Delta a_2|+|\Delta a_3|+|\Delta a_4|+|\alpha a_5|}{5}$

$=\frac{0.01+0.06+0.09+0.18+0.20}{5}=\frac{0.54}{5}=0.108=0.11\text{sec}$

3. **(c)**

Here, $\frac{dF}{F}\times 100=4\%$ and $\frac{dl}{l}\times 100=2\%$

As $P=\frac{F}{l^2}$, $\frac{dP}{P}=\frac{dF}{F}+\frac{2dl}{l}$

Maximum % error in $P=\frac{dP}{P}\times 100=\frac{dF}{F}\times 100+\frac{2dl}{l}\times 100$

$=4\%+2\times 2\%=8\%$

4. **(d)**

Here, least count = 0.001 cm

Main scale reading = 5 mm = 0.5 cm

Circular scale reading = 25

Zero error = – 0.004 cm

Observed diameter of steel ball $= MSR + CSR \times LC$

$$= 0.5\,cm + 25 \times 0.001 = 0.525\,cm$$

Corrected diameter of steel ball = observed diameter + zero correction

$$= 0.525\,cm + 0.004\,cm = 0.529\,cm$$

5. **(d)**

Position vector of the origin of the rectangular coordinate system is a null vector and displacement vector of a stationary object is a null vector.

6. **(a)**

Knowledge based question.

7. **(c)**

$\vec{A} = 4\hat{i} + 3\hat{j} + 12\hat{k};\ A = \sqrt{4^2 + 3^2 + 12^2} = 13$

Let θ be the angle which $\vec{A}$ makes with $x-$axis, then

$$\cos\theta = \frac{A_x}{A} = \frac{4}{13} \text{ or } \theta = \cos^{-1}\left(\frac{4}{13}\right)$$

8. **(d)**

Since, $\vec{C} \perp^r$ to $\vec{A}$, therefore

$B^2 = A^2 + C^2 = A^2 + A^2 = 2A^2$ or $B = \sqrt{2}$ A.

Given $\vec{A} + \vec{B} = \vec{C}$

$\therefore\ A^2 + B^2 + 2AB\cos\theta = C^2 = A^2$ $\quad (\because C = A)$

or $A^2 + \left(\sqrt{2}A\right)^2 + 2A\left(\sqrt{2}A\right)\cos\theta = A^2$

or $\cos\theta = -\frac{1}{\sqrt{2}} = \cos\frac{3\pi}{4}$ or $\theta = \frac{3\pi}{4}$

9. **(c)**

When particle is moving with uniform acceleration let v be the velocity of particle at a distance s, then average velocity

$$= \frac{0 + v}{2} = v/2$$

time taken $t_1 = \frac{s}{(v/2)} = \frac{2s}{v}$

When particle moves with uniform velocity, time taken, $t_2 = \frac{2s}{v}$

When particle moves with uniform retardation, time taken,

$$t_3 = \frac{3s}{(0+v)/2} = \frac{6s}{v}$$

Total time $= t_1 + t_2 + t_3 = \frac{2s}{v} + \frac{2s}{v} + \frac{6s}{v} = \frac{10s}{v}$

$$\therefore v_{av} = \frac{s+2s+3s}{10s/v} = \frac{6v}{10} \therefore \frac{v_{av}}{v} = \frac{6}{10} = \frac{3}{5}$$

10. **(b)**

Using the relation, $S = ut + \frac{1}{2}at^2$, we have

$$h = u\cos\theta t_1 - \frac{1}{2}gt_1^2 = u\cos\theta t_2 - \frac{1}{2}gt_2^2$$

or $u\cos\theta \times 1 - \frac{1}{2}\times 9.8 \times 1^2 = u\cos\theta \times 3 - \frac{1}{2}\times 9.8\times 3^2$

or $u\cos\theta(3-1) = 4.9\times(9-1) = 4.9\times 8$

$$u\cos\theta = \frac{4.9\times 8}{2} = 4.9\times 4 = 19.6\,m/s$$

Max. height $= \frac{u^2\cos^2\theta}{2g} = \frac{(19.6)^2}{2\times 9.8} = 19.6\,m$

11. **(d)**

In circular motion, the centripetal force always acts perpendicular to the velocity or displacement vector.

Hence workdone, $W = \vec{F}.\vec{S} = FS\cos 90^\circ = 0$

Which is true what ever may be the displacement of body on circular path.

12. **(b)**

Range is maximum when $\theta = 45^\circ$. Maximum range, $R_{\max} = \frac{u^2}{g} = 12km$

Initial momentum at the highest point of mass

$$= mu\cos 45^\circ = mu/\sqrt{2}$$

After explosion, the projectile breaks into two equal masses. One mass $(= m/2)$ drops vertically downwards. Hence its velocity and momentum is zero.

If v is the velocity of second mass then following the law of conservation of linear momentum, we have

$$mu\cos 45^\circ = \frac{mv}{2} \text{ or } v = 2u\cos 45^\circ$$

Time taken by second mass to go from highest point to ground,

$$t=\frac{u\sin\theta}{g}=\frac{u\sin 45^{\circ}}{g}$$

Horizontal distance covered by the second mass from the time of explosion up to ground is

$$=vt=2u\ \cos 45^{\circ}\times\frac{u\sin 45^{\circ}}{g}$$

$$=\frac{u^2\sin 2\times 45^{\circ}}{g}=\frac{u^2}{g}=\mathbf{12km}$$

13. **(c)**

Change in linear momentum between 0 and 7 s is

$=$algebraic sum of areas enclosed from 0 to 7s is

$=-$ area $OAB+$ area BCDE – area EFG + area GHU – area JKL

$$=-\frac{1\times 1}{2}+2\times 2-\frac{2\times 2}{2}+1\times 1-\frac{1\times 1}{2}=2\,\mathbf{N\text{-}s}$$

14. **(d)**

Here, $m=3\,kg, u=10\,m/s, \theta=60^{\circ}, t=0.20\,s, F=?$

As $F\times t=$ change in momentum of the ball

$$=mv\sin\theta-(-m\ v\ \sin\ \theta)=2mv\sin\theta$$

$$f=\frac{2mv\ \sin\theta}{t}=\frac{2\times 3\times 10\sin 60^{\circ}}{0.2}=150\sqrt{3}\mathbf{N}$$

15. **(b)**

$$m=2kg, v_x=3\,m/s, f_y=4\text{N}, \text{a}_\text{y}=\frac{f_y}{m}=\frac{4}{2}=2\,m/s^2$$

$$s_y=u_yt+\frac{1}{2}a_yt^2=0+\frac{1}{2}\times 2\times 4^2=16\,m$$

$$s_x=u_x\times t=3\times 4=12\,m$$

$$s=\sqrt{s_x^2+s_y^2}=\sqrt{12^2+16^2}=\mathbf{20m}$$

16. **(c)**

From $s=ut+\frac{1}{2}at^2=\frac{1}{2}at^2=\frac{1}{2}(\mu g)t^2$

$$t=\sqrt{\frac{2x}{\mu g}}\quad \therefore t\propto\frac{1}{\sqrt{\mu}}$$

17. **(d)**

As is clear from Fig.

$$\phi_A + \phi_B + \phi_C = \frac{q}{\epsilon_0}$$

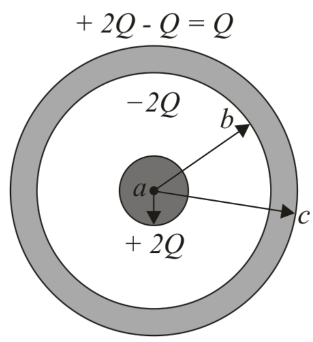
$$2\phi_A = \frac{q}{\epsilon_0} - \phi_B = \frac{q}{\epsilon_0} - \phi$$

$$\phi_A = \frac{1}{2}\left(\frac{q}{\epsilon_0} - \phi\right)$$

18. **(a)**

In figure $+2Q$ is charge on inner sphere of radius a. By induction, $-2Q$ charge appears on inner surface of radius b and $+2Q$ charge appears on outer surface of radius c. As outer surface has charge $-Q$, therefore has charge $-Q$, therefore total charge on outer surface of shell $= 2Q - Q = Q$.

$\therefore$ Surface charge density on inner surface of shell $= -2Q/4\pi b^2$, and surface charge density on outer surface of shell $= Q/4\pi b^2$.



Choice (a) is correct.

19. **(b)**

Here, $K.E. = 100\,eV = 100 \times 1.6 \times 10^{-19}$ joule. This is lost when electron moves through a distance *(d)* towards the negative plate.

K.E. $=$ work done $= F \times s = qE \times s = e\left(\frac{\sigma}{\epsilon_0}\right)d$

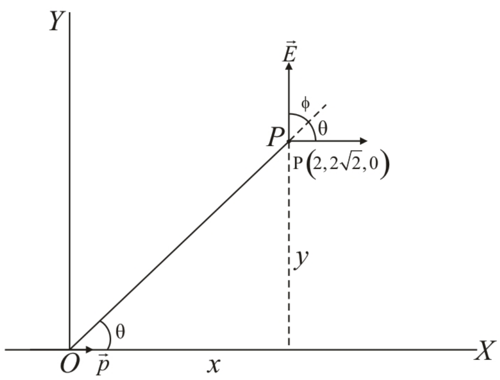
$$d = \frac{(K.E.)\epsilon_0}{e\sigma} = \frac{100 \times 1.6 \times 10^{-19} \times 8.86 \times 10^{-12}}{1.6 \times 10^{-19} \times 2 \times 10^{-6}} = 4.43 \times 10^{-4}\,m = \mathbf{0.443\,mm}$$

20. **(d)**

Let $P(2, 2\sqrt{2}, 0)$ be the given point. As is clear from fig.

$1(FPH)6\ \ x = 2,\ y = 2\sqrt{2}, z = 0$

$$\tan\theta = \frac{y}{x} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$



$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\sqrt{2}}$

Let the electric field $\vec{E}$ due to the dipole be at angle ϕ with OP.

$\therefore\ \tan\phi = \dfrac{1}{2}\tan\theta = \dfrac{1}{2}.\sqrt{2} = \dfrac{1}{\sqrt{2}} = \cos\theta$

$\therefore\ \phi = 90^\circ - \theta$

21. **3**

Here, $\vec{A} = 2\hat{i} - 2\hat{j} - 2\hat{j} + \hat{k};$

$\hat{A} = \dfrac{\hat{A}}{A} = \dfrac{2\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{2^2 + (-2)^2 + 1^2}} = \dfrac{2\hat{i} - 2\hat{j} + \hat{k}}{3}$

$\vec{B} = 2\hat{i} + 2\hat{j} + \hat{k}$

The component of $\vec{B}$ in the direction of $\vec{A}$ is

$= \vec{B}.\hat{A} = \left(2\hat{i} + 2\hat{i} + 2\hat{j} + \hat{k}\right)\dfrac{\left(2\hat{i} - 2\hat{j} + \hat{k}\right)}{3} = \dfrac{4-4+1}{3} = \dfrac{1}{3}$

22. **5.25**

Here, $u = 150\,ms^{-1}; \theta = 60^\circ, t = ?\ \beta = 45^\circ$

Let v_x and u_y be component velocity of projectile after time t, when $\beta = 45^\circ$. Then

$u_x = u_x \sin 60^\circ = 150 \times \dfrac{1}{2} = 75\,ms^{-1}$

$u_y = u_x \sin 60^\circ - g\,t = 150 \times \sqrt{3}/2 - 10 \times t = 75\sqrt{3} - 10t$

$\tan\beta = \dfrac{u_y}{u_x}$ or $1 = \dfrac{u_y}{u_x}$ or $u_x = u_y$

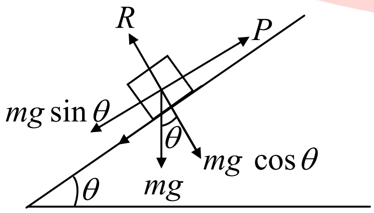
or $75 = 75\sqrt{3} - 10t$ or $10t = 75\sqrt{3} - 75 = 75\left(\sqrt{3} - 1\right)$

or $t = 75 \times (1.732 - 1)/10 = 5.49s$

23. **98**

Here, $m = 10\,kg, \theta = 30^\circ, \mu = 0.5$

As is clear from Fig. minimum force required just to pull the body up the plane is



P = $mg\sin\theta + f = mg\sin\theta + \mu R = mg\theta + \mu mg\cos\theta$

$= mg(\sin\theta + \beta\cos\theta)$

$= 10\times 9.8(\sin 30^o + 0.5\cos 30^o) = \mathbf{91.4N}$

24. **2.50**

Total mass $m = 7 + 5 = 12$ kg

$\therefore$ Common acceleration $a = \frac{F}{m} = \frac{6}{12} = 0.5\, m/s^2$

Force on lighter block $= 5\times a = 5\times 0.5 = \mathbf{2.5N}$

25. **0.8**

If q, q' are the charge on spheres of radii R and 2 R respectively, then as $\sigma = \sigma'$

$\frac{q}{4\pi R^2} = \frac{q}{4\pi(2R)^2}$ or $q' = 4g$

As $q + q' = Q, q + 4q = \frac{Q}{5}$

$q' = 4q = \frac{4Q}{5}$

CHEMISTRY

26. **(c)**	27. **(c)**	28. **(c)**	29. **(b)**
30. **(a)**	31. **(c)**	32. **(d)**	

33. **(a)**

$MgCO_3 \rightarrow MgO + CO_2$. Thus 84 g $MgCO_3$ give 22.4 L CO_2 at S.T.P. Hence 8.4g will give 2.24 L CO_2 at S.T.P.

34. **(a)**

Net reaction is :

A + 2Z + C $\rightarrow$ Z + F

As per the given information Z is the L. R.

$\therefore$ The no. of moles of F produced = $\frac{1}{2} \times 6 = 3$

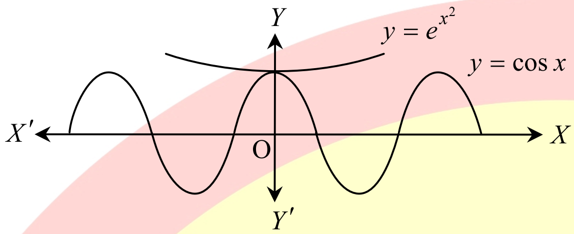
35. **(a)**	36. **(a)**	37. **(a)**	38. **(c)**
39. **(d)**	40. **(a)**	41. **(a)**	42. **(d)**
43. **(a)**	44. **(a)**	45. **(b)**	46. **(7)**
47. **(4)**	48. **(7)**	49. **(6)**	50. **(7)**

MATHS

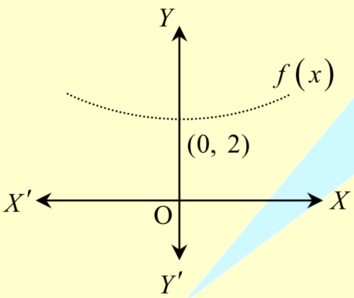
51. **(d)**

$f: R \to R$

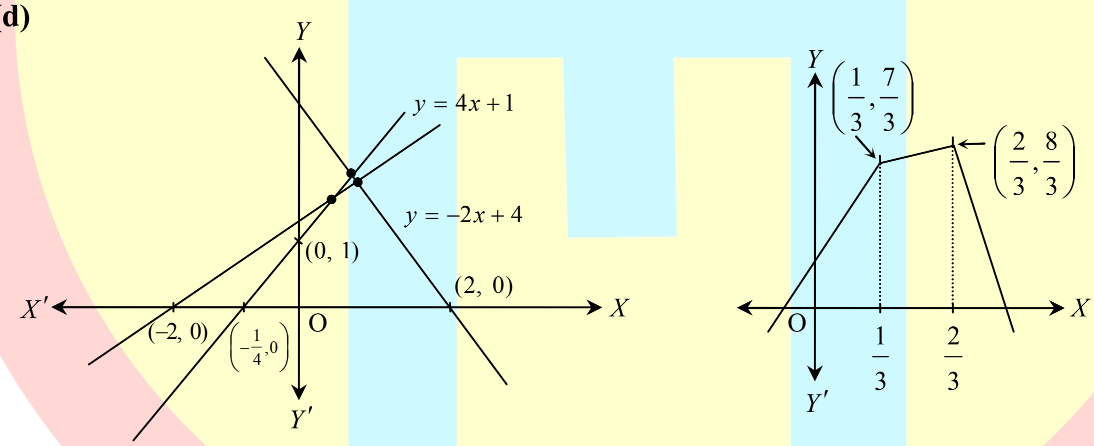
$f(x) = e^{x^2} + \cos x$ (even function) hence many-one.



From graph, it is clear that $f(x)$ can't take negative values. Hence into function.



52. **(d)**



Solving $y = 4x + 1;\ y = -2x + 4$

$4x + 1 = -2x + 4$

$6x = 3;\ x = \frac{1}{2}$ at $x = \frac{1}{2},\ y = 3$

Solving $y = x + 2;\ y = -2x + 4$

$x + 2 = -2x + 4;\ 3x = 2;\ x = \frac{2}{3}$

at $x = \frac{2}{3},\ y = \frac{8}{3}$

Solving, $y = x + 2$ and $y = 4x + 1$

$4x + 1 = x + 2;\ 3x = 1;\ x = \frac{1}{3}$

at $x = \frac{1}{3};\ y = \frac{7}{3}$

Thus maximum $f(x) = \frac{8}{3}$

53. **(a)**

Here, $-1 \le x \le 1 \Rightarrow -\frac{\pi}{4} \le \tan^{-1} x \le \frac{\pi}{4}$

$\Rightarrow -\frac{\pi}{2} \le 2\tan^{-1} x \le \frac{\pi}{2}$

For, $x^2 < 1$ so $-1 < x < 1$

$f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1} x$

Hence, function is one-one and onto.

54. **(b)**

Given, function is $y = 3x - 5$

$$x = \frac{y+5}{3}$$

Interchange x and y

$y = \frac{x+5}{3} \Rightarrow f^{-1}(x) = \frac{x+5}{3}$

55. **(b)**

$$\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{\frac{\left(\frac{a}{x}+\frac{b}{x^2}\right)}{\left(\frac{a}{x}+\frac{b}{x^2}\right)}2x\cdot} = e^{2a}\left[\text{as } \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\right]$$

$e^{2a} = e^2 \Rightarrow a = 1,\ b \in R$

56. **(d)**

$$\lim_{x\to 0}\left(1 + x\ln\left(1+b^2\right)\right)^{1/x}\left(1^\infty\right) = \lim_{x\to 0}\left(1 + x\ln\left(1+b^2\right)\right)^{\frac{1}{x\ln\left(1+b^2\right)}\ln\left(1+b^2\right)}$$

$= e^{\ln\left(1+b^2\right)} = 1 + b^2$

$1 + b^2 = 2b\sin^2\theta$ (given)

$1 + b^2 = 2b\left(1 - \cos^2\theta\right)$

$1 + b^2 = 2b - 2b\cos^2\theta$

$$(1-b)^2+2b\cos^2\theta=0$$

As $b>0$; so $1-b=0$; $\cos\theta=0$

$$b=1;\quad \theta=\pm\frac{\pi}{2}$$

57. **(d)**

$$L=\lim_{x\to\infty}\frac{1}{x^6}\int_0^{x^2}\frac{(t^6+5t^4+55)}{(2t^4+t+5)}dt$$

$$L=\lim_{x\to\infty}\frac{1}{6x^5}\frac{(x^{12}+5x^8+55)}{(2x^8+x^2+5)}2x=\lim_{x\to\infty}\frac{2}{6x^4}\frac{(x^{12}+5x^8+55)}{(2x^8+x^2+5)}=\frac{2}{6}\cdot\frac{1}{2}=\frac{1}{6}$$

58. **(b)**

$$\text{R.H.L.}=\lim_{x\to0^+}\frac{\left(\sqrt{(x+x^2)}-\sqrt{x}\right)\left(\sqrt{x+x^2}+\sqrt{x}\right)}{x^{3/2}\left(\sqrt{(x+x^2)}+\sqrt{x}\right)}$$

$$=\lim_{x\to0^+}\frac{x^2}{x^{3/2}\left(\sqrt{(x+x^2)}+\sqrt{x}\right)}=\lim_{x\to0^+}\frac{x^2}{x^{3/2}\left(\sqrt{(1+x)}+1\right)}=\frac{1}{2}$$

$$\text{L.H.L.}=\lim_{x\to0^-}\frac{\sin(p+1)x+\sin x}{x}=\lim_{x\to0^-}\frac{2\sin\left(\left(\frac{p+2}{2}\right)x\right)\left(\frac{px}{2}\right)}{x}=p+2$$

$$p+2=\frac{1}{2};\ p=-\frac{3}{2};\ q=\frac{1}{2}$$

59. **(d)**

$$(2)^2=\lim_{x\to2}\frac{k(x-2)(x+2)}{(2-x)}=-4k$$

$k=-1$ only

60. **(a)**

$$(1)^2+p\cdot1+1=p+2+q$$

$$q=0$$

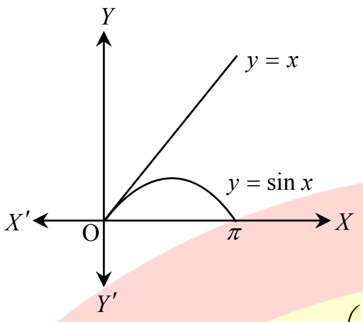
$$4+2p+1=4p+4+q$$

$$2p+q=1;\ p=\frac{1}{2}$$

$$p+q=\frac{1}{2}+0=\frac{1}{2}$$

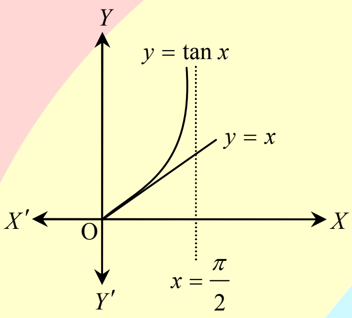
61. **(c)**

$\sin x = x$



From graph, $\sin x = x$ has no root in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

$\tan x = x$



It has no root in $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$.

It is true that $f(x)$ is continuous in [0, 2] so it will be bounded. Thus $f(x) \geq c$ for all $x \in [0, 2]$. Thus option (c) is correct. Option (d) is incorrect because $g(x)$ is not a continuous function.

62. **(d)**

$\lim_{x\to 1^-} f(x) = 1$

$\lim_{x\to 1^+} f(x) = a + b$

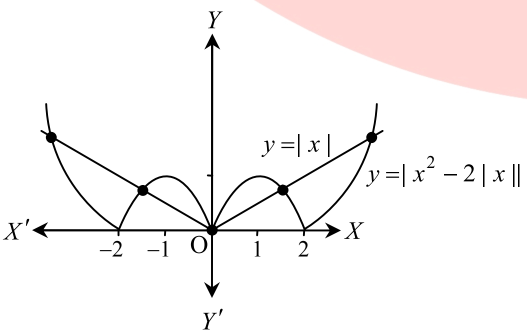
$1 = a + b$ Also $f(1) = 1$

$\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} 2xe^{2(x-1)} + x^2e^{2(x-1)}2 = 4$

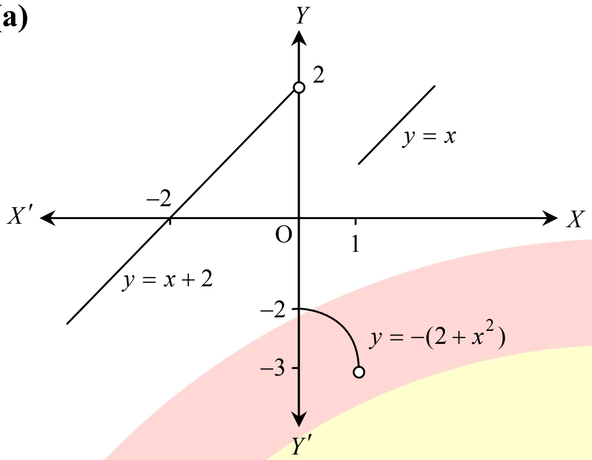
$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} -2a\sin(2(x-1)) + 2bx = 2b$

Here $2b = 4$; $b = 2$; $a = -1$

63. **(d)**



64. **(a)**



$|f(x)|$ is not derivable at $x = -2, 0, 1$.

65. **(a)**

$$f'(x)\Big|_{\text{at } x=k} = \lim_{x\to k}\frac{f(x)-f(k)}{x-k} = \lim_{h\to 0^+}\frac{f(k-h)-f(k)}{-h}$$

$$= \lim_{h\to 0^+}\frac{[k-h]\sin(\pi(k-h))-[k]\sin\pi k}{-h} = \lim_{h\to 0}\frac{(k-1)(-1)^{k+1}\sin\pi h}{-h}$$

$$= \lim_{h\to 0}(k-1)(-1)^k\pi = (-1)^k(k-1)\pi$$

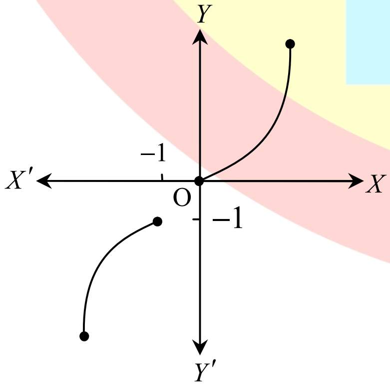
66. **(d)**

$$f(x) = \begin{cases}\sin x - x; & x \ge 0\\ -\sin x + x; & x < 0\end{cases} \Rightarrow f'(x) = \begin{cases}\cos x - 1; & x \ge 0\\ -\cos x + 1; & x < 0\end{cases}$$

Thus, at $x = 0$, $f(x)$ is differentiable.

67. **(d)**

$$g(x) = \begin{cases}2-3x^2; & -\infty < x \le -1\\ mx; & -1 < x < 0\\ x^2; & 0 \le x < \infty\end{cases}$$



At $x = -1$

$-1 \le -m \Rightarrow m \le 1$

Also, $m > 0$

68. **(c)**

$$\frac{x^2}{c}+\frac{y^2}{4}=1;\ \ y^3=16x;$$

$$\frac{2x}{c}+\frac{2y}{4}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{4x}{cy}$$

$$m_1=-\frac{4x}{cy}$$

$$3y^2\frac{dy}{dx}=16$$

$$\frac{dy}{dx}=\frac{16}{3y^2}=m_2$$

$$m_2=\frac{16}{3y^2}$$

$$m_1m_2=-1$$

$$-\frac{4x}{cy}\cdot\frac{16}{3y^2}=-1$$

$$\frac{64}{3c}\cdot\frac{x}{y^3}=1$$

$$\frac{64}{3c}\cdot\frac{1}{16}=1$$

$$c=\frac{4}{3}$$

69. **(d)**

$f(x)=|x|$ not derivable at $x=0$

$f(x)=\tan x$ not continuous at $x=\frac{\pi}{2}$

$$f(x)=1+(x-2)^{2/3}$$

$f'(x)=\frac{2}{3(x-2)^{1/3}}$ not derivable at $x=2$

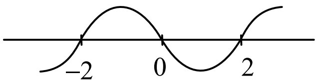
$f(x)=x(x-2)^2$ in $x\in[0,\ 2]$ Rolle's theorem is valid

70. **(b)**

$$f(x)=\int_3^{x^2}e^{-t^2}(t-4)(t^2-t+12)\,dt$$

$f'(x)=e^{-x^4}\left(x^2-4\right)\left(x^4-x^2+12\right)2x$

$=e^{-x^4}(x-2)(x+2)\left(x^4-x^2+12\right)x<0$



$x\in(-\infty,-2)\cup(0,2)$

71. **(3)**

$y=\dfrac{x^2+x+7}{x+2};\ x\in R;\ x\neq -2$

$x^2+(1-y)x+(7-2y)=0;\ x\in R$

$D=(1-y)^2-4(7-2y)\geq 0$

$1+y^2-2y-28+8y\geq 0$

$y^2+6y-27\geq 0\ \Rightarrow (y+9)(y-3)\geq 0$

$y\in(-\infty,-9]\cup[3,\infty)$

$y=3$

72. **(2)**

$\left[x^2\right]+[x-2]<0\ \Rightarrow [x]^2+[x]-2<0$

$[x]^2+2[x]-[x]-2<0\ \Rightarrow ([x]+2)([x]-1)<0$

$\Rightarrow -2<[x]<1\ \Rightarrow -1\leq [x]\leq 0$

$\Rightarrow -1\leq x<1$ and $\{x\}=\dfrac{1}{2};\ x-[x]=\dfrac{1}{2}$

Case I : $-1\leq x<0;\ [x]=-1;\ x-[x]=\dfrac{1}{2}$

$x+1=\dfrac{1}{2};\ x=-\dfrac{1}{2}(\text{valid})$

Case II : $0\leq x<1;\ [x]=0;\ x-0=\dfrac{1}{2};\ x=\dfrac{1}{2}(\text{valid})$

73. **(1)**

Since, $f(x)$ is a positive increasing function

$\Rightarrow\ 0<f(x)<f(2x)<f(3x)$

$\Rightarrow\ 0<1<\dfrac{f(2x)}{f(x)}<\dfrac{f(3x)}{f(x)}$

$$\Rightarrow \lim_{x\to\infty} 1 \le \lim_{x\to\infty} \frac{f(2x)}{f(x)} \le \lim_{x\to\infty} \frac{f(3x)}{f(x)}$$

By sandwich theorem, $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$

74. **(7)**

$$\lim_{x\to 2} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2} = k$$

$$\lim_{x\to 2} \frac{(x-2)^2 (x+5)}{(x-2)^2} = \lim_{x\to 2} (x+5) = 7$$

So, $k = 7$

75. **(3)**

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \left(\frac{x\sin x + 2\cos 2x}{2} \right)^{\frac{1}{x^2}}$$

$$= \lim_{x\to 0} \left(1 + \frac{x\sin x + 2\cos^2 x - 2}{2} \right)^{\frac{1}{\frac{x\sin x + 2\cos 2x - 2}{2}} \cdot \frac{x\sin x + 2(\cos 2x - 1)}{2x^2}}$$

$$= e^{\lim_{x\to 0} \frac{\frac{\sin x}{x} - 4\frac{\sin^2 x}{x^2}}{2}} = e^{-\frac{3}{2}} = e^{\frac{-k}{2}}$$

So, $k = 3$